# Implicit Force Control with only joint measurements 

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#### Abstract

One of the main practical problems on cooperative robots is the complexity of integrating a large amount of expensive velocity-force sensors. In this paper, the control of cooperative robots using only joint measurements is considered to manipulate an object firmly. Experimental results are shown to support the developed theory.


Key words: Cooperative robot systems, observer design.

## 1 INTRODUCTION

In cooperation control the tasks characterized by physical contact between the end effector and a constraint surface are particularly interesting. A long list of such tasks can be given: scribing, writing, deburring, grinding, etc. To control two or more cooperative robots, there have been proposed mainly three kinds of approaches: masterslave model, centralized controller and decentralized architecture controller. In the decentralized architecture there is no need to handle highdimensional matrices. Furthermore, the control laws for all the robots are the same, so its implementation is straightforward. For example, Liu et al. (1997) proposed a model-based adaptive control for cooperation of robots.

Recently, there has been a considerable the development of nonlinear controllers for robot

[^0]manipulators focused on reducing the number of sensors required to implement the control algorithm. However, the literature available on cooperative robots not requiring link velocity and end-effector force measurements is very limited. In Huang and Tzeng (1991), two types of force observers are designed for constrained robot systems. In de Queiroz et al. (1997), an adaptive approach is proposed, which does not require velocity measurements. This approach, however, has the disadvantage that a transformation has to be accomplished on line. Liu and Arimoto (1996) proposed a simple controller without force feedback by using the joint-space orthogonalization scheme, which decouples position and force signals in the joint spaces. However, their approach still needs velocity measurement and no experimental results are presented.

In this paper, their method is used to design a decentralized position-force tracking controller for cooperative robot systems which does not require link velocities measurements nor end-
effector contact forces. The approach is based on that presented in Gudiño Lau et al. (2004). However, two important improvements over the original algorithm are introduced: the observer is much simpler and, just as mentioned, no force measurements are required.

The paper is organized as follows. In Section 2 , the system model and its properties are presented. Section 3 describes the proposed control and observer laws, while Section 4 shows experimental results. Finally, Section 5 gives some conclusions.

## 2 SYSTEM MODEL AND PROPERTIES

Consider a cooperative system with $l$-fingers, each of them with $n_{i}$ degrees of freedom and $m_{i}$ constraints arising from the contact with a held object. Then, the total number of degrees of freedom is given by $n=\sum_{i=1}^{l} n_{i}$ with a total number of $m=\sum_{i=1}^{l} m_{i}$ constraints, where $n_{i}>m_{i}$. The dynamics of the $i$-th finger is given by (Parra-Vega et al. 2001)

$$
\begin{align*}
\boldsymbol{\tau}_{i}+\boldsymbol{J}_{\varphi_{i}}^{\mathrm{T}}\left(\boldsymbol{q}_{i}\right) \boldsymbol{\lambda}_{i} & =\boldsymbol{H}_{i}\left(\boldsymbol{q}_{i}\right) \ddot{\boldsymbol{q}}_{i}+\boldsymbol{C}_{i}\left(\boldsymbol{q}_{i}, \dot{\boldsymbol{q}}_{i}\right) \dot{\boldsymbol{q}}_{i}  \tag{1}\\
& +\boldsymbol{D}_{i} \dot{\boldsymbol{q}}_{i}+\boldsymbol{g}_{i}\left(\boldsymbol{q}_{i}\right)
\end{align*}
$$

where $\boldsymbol{q}_{i} \in \mathbb{R}^{n_{i}}$ is the vector of generalized joint coordinates, $\boldsymbol{H}_{i}\left(\boldsymbol{q}_{i}\right) \in \mathbb{R}^{n_{i} \times n_{i}}$ is the symmetric positive definite inertia matrix, $\boldsymbol{C}_{i}\left(\boldsymbol{q}_{i}, \dot{\boldsymbol{q}}_{i}\right) \dot{\boldsymbol{q}}_{i} \in \mathbb{R}^{n_{i}}$ is the vector of Coriolis and centrifugal torques, $\boldsymbol{g}_{i}\left(\boldsymbol{q}_{i}\right) \in \mathbb{R}^{n_{i}}$ is the vector of gravitational torques, $\boldsymbol{D}_{i} \in \mathbb{R}^{n_{i} \times n_{i}}$ is the positive semidefinite diagonal matrix accounting for joint viscous friction coefficients, $\boldsymbol{\tau}_{i} \in \mathbb{R}^{n_{i}}$ is the vector of torques acting at the joints, and $\boldsymbol{\lambda}_{i} \in \mathbb{R}^{m_{i}}$ is the vector of Lagrange multipliers (physically represents the force applied at the contact point). $\boldsymbol{J}_{\varphi_{i}}\left(\boldsymbol{q}_{i}\right)=\boldsymbol{\nabla} \boldsymbol{\varphi}_{i}\left(\boldsymbol{q}_{i}\right) \in \mathbb{R}^{m_{i} \times n_{i}}$ is assumed to be full rank in this paper. $\nabla \boldsymbol{\varphi}_{i}\left(\boldsymbol{q}_{i}\right)$ denotes the gradient (or the Jacobian matrix) of the object surface vector $\varphi_{i} \in \mathbb{R}^{m_{i}}$ which maps a vector onto the normal plane at the tangent plane that arises at the contact point described by
$\boldsymbol{\varphi}_{i}\left(\boldsymbol{q}_{i}\right)=\mathbf{0}$.

Note that equation (2) means that homogeneous constraints are being considered (Parra-Vega et al. 2001). The complete system is subjected to $m$ holonomic constraints given by
$\varphi(\boldsymbol{q})=\mathbf{0}$,
where $\boldsymbol{\varphi}(\boldsymbol{q})=\boldsymbol{\varphi}\left(\boldsymbol{q}_{1}, \ldots, \boldsymbol{q}_{l}\right) \in \mathbb{R}^{m}$. This means that the object being manipulated and the environment are modeled by the constraint (3).

Let us denote the largest (smallest) eigenvalue of a matrix by $\lambda_{\max }(\cdot)\left(\lambda_{\min }(\cdot)\right)$. By recalling that revolute joints are considered, some properties can be established (Gudiño Lau et al. 2004); since most of them are well known, here we present only the following.

Property 5 The vector $\dot{\boldsymbol{q}}_{i}$ can be written as
$\dot{\boldsymbol{q}}_{i} \triangleq \boldsymbol{Q}_{i}\left(\boldsymbol{q}_{i}\right) \dot{\boldsymbol{q}}_{i}+\boldsymbol{J}_{\varphi_{i}}^{+}\left(\boldsymbol{q}_{i}\right) \dot{\boldsymbol{p}}_{i}$,
where $\boldsymbol{J}_{\varphi_{i}}^{+}=\boldsymbol{J}_{\varphi_{i}}^{\mathrm{T}}\left(\boldsymbol{J}_{\varphi_{i}} \boldsymbol{J}_{\varphi_{i}}^{\mathrm{T}}\right)^{-1} \in \mathbb{R}^{n_{i} \times m_{i}}$ stands for the Penrose's pseudoinverse and $\boldsymbol{Q}_{i} \in \mathbb{R}^{n_{i} \times n_{i}}$ satisfies $\operatorname{rank}\left(\boldsymbol{Q}_{i}\right)=n_{i}-m_{i}$. These two matrices are orthogonal, i.e. $\boldsymbol{Q}_{i} \boldsymbol{J}_{\varphi_{i}}^{+}=\boldsymbol{O}\left(\right.$ and $\boldsymbol{Q}_{i} \boldsymbol{J}_{\varphi_{i}}^{\mathrm{T}}=$ $\boldsymbol{O}) . \dot{\boldsymbol{p}}_{i} \triangleq \boldsymbol{J}_{\varphi_{i}} \dot{\boldsymbol{q}}_{i} \in \mathbb{R}^{m_{i}}$ is the so called constrained velocity. Since homogeneous constraints are being considered, it also holds in view of (2) that
$\dot{\boldsymbol{p}}_{i}=\mathbf{0} \quad$ and $\quad \boldsymbol{p}_{i}=\mathbf{0}$,
for $i=1, \ldots, l . \boldsymbol{p}_{i}$ is called the constrained position.

To be able to design the control-observer scheme, the following assumptions are made.

Assumption 1 The l robots of which the system is made up satisfy constraints (2) and (4) for all time. Furthermore, none of the robots is redundant nor it is in a singularity.

Assumption 2 The matrix $\boldsymbol{J}_{\varphi_{i}}$ is Lipschitz continuous, i. e. $\left\|\boldsymbol{J}_{\varphi_{i}}\left(\boldsymbol{q}_{i}\right)-\boldsymbol{J}_{\varphi_{i}}\left(\boldsymbol{q}_{\mathrm{d} i}\right)\right\| \leq L_{i}\left\|\boldsymbol{q}_{i}-\boldsymbol{q}_{\mathrm{d} i}\right\|$, for a positive constant $L_{i}$ and for all $\boldsymbol{q}_{i}, \boldsymbol{q}_{\mathrm{d} i} \in$ $\mathbb{R}^{n_{i}}$. Besides, there exist positive finite constants $c_{0 i}$ and $c_{1 i}$ which satisfies
$c_{0 i} \triangleq \max _{\forall \mathbf{q}_{i} \in \mathbb{R}^{n_{i}}}\left\|\boldsymbol{J}_{\varphi_{i}}^{+}\left(\boldsymbol{q}_{i}\right)\right\|$
$c_{1 i} \triangleq \max _{\forall \mathbf{q}_{i} \in \mathbb{R}^{n_{i}}}\left\|\boldsymbol{J}_{\varphi_{i}}\left(\boldsymbol{q}_{i}\right)\right\|$.

## 3 CONTROL WITH VELOCITY ESTIMATION

## Control Law

Consider model (1) and define the tracking and observation errors as $\tilde{\boldsymbol{q}}_{i} \triangleq \boldsymbol{q}_{i}-\boldsymbol{q}_{\mathrm{d} i}$ and $\boldsymbol{z}_{i} \triangleq \boldsymbol{q}_{i}-$ $\hat{\boldsymbol{q}}_{i}$, where $\boldsymbol{q}_{\mathrm{d} i}$ is a desired smooth bounded trajectory satisfying constraint (2), and ( $\hat{\bullet}$ ) represents the estimated value of $(\cdot)$. Other error definitions are $\Delta \boldsymbol{p}_{i} \triangleq \boldsymbol{p}_{i}-\boldsymbol{p}_{\mathrm{d} i}$ and $\Delta \boldsymbol{\lambda}_{i} \triangleq \boldsymbol{\lambda}_{i}-\boldsymbol{\lambda}_{\mathrm{d} i}$, where $\boldsymbol{p}_{\mathrm{d} i}$ is the desired constrained position which satisfies (4). $\boldsymbol{\lambda}_{\mathrm{d} i}$ is the desired force to be applied by each finger on the constrained surface. Also

$$
\begin{align*}
\dot{\boldsymbol{q}}_{\mathrm{r} i} & \triangleq \boldsymbol{Q}_{i}\left(\boldsymbol{q}_{i}\right)\left(\dot{\boldsymbol{q}}_{\mathrm{d} i}-\boldsymbol{\Lambda}_{i}\left(\hat{\boldsymbol{q}}_{i}-\boldsymbol{q}_{\mathrm{d} i}\right)\right)  \tag{7}\\
& +\boldsymbol{J}_{\varphi_{i}}^{+}\left(\boldsymbol{q}_{i}\right)\left(\dot{\boldsymbol{p}}_{\mathrm{d} i}-\beta_{i} \Delta \boldsymbol{p}_{i}\right) \\
\boldsymbol{s}_{i} & \triangleq \dot{\boldsymbol{q}}_{i}-\dot{\boldsymbol{q}}_{\mathrm{r} i} \triangleq \boldsymbol{s}_{\mathrm{p} i}+\boldsymbol{s}_{\mathrm{f} i} \tag{8}
\end{align*}
$$

where $\boldsymbol{\Lambda}_{i}=k_{i} \boldsymbol{I} \in \mathbb{R}^{n_{i} \times n_{i}}$ with $k_{i}>0$, is a diagonal positive definite matrix, and $\beta_{i}$ is a positive constant. Note also that $\boldsymbol{s}_{\mathrm{p} i}$ and $\boldsymbol{s}_{\mathrm{f} i}$ are orthogonal vectors. We propose the following substitution for $\ddot{\boldsymbol{q}}_{\mathrm{r} i}$

$$
\begin{align*}
\ddot{\hat{\boldsymbol{q}}}_{\mathrm{r} i} & \triangleq \boldsymbol{Q}_{i}\left(\boldsymbol{q}_{i}\right)\left(\ddot{\boldsymbol{q}}_{\mathrm{d} i}-\boldsymbol{\Lambda}_{i}\left(\dot{\hat{\boldsymbol{q}}}_{i}-\dot{\boldsymbol{q}}_{\mathrm{d} i}\right)\right)  \tag{9}\\
& +\boldsymbol{J}_{\varphi_{i}}^{+}\left(\boldsymbol{q}_{i}\right)\left(\ddot{\boldsymbol{p}}_{\mathrm{d} i}-\beta_{i}\left(\dot{\boldsymbol{p}}_{i}-\dot{\boldsymbol{p}}_{\mathrm{d} i}\right)\right) \\
& +\dot{\hat{\boldsymbol{Q}}}_{i}\left(\dot{\boldsymbol{q}}_{\mathrm{o} i}\right)\left(\dot{\boldsymbol{q}}_{\mathrm{d} i}-\boldsymbol{\Lambda}_{i}\left(\hat{\boldsymbol{q}}_{i}-\boldsymbol{q}_{\mathrm{d} i}\right)\right) \\
& +\dot{\hat{\boldsymbol{J}}}_{\varphi_{i}}^{+}\left(\dot{\boldsymbol{q}}_{\mathrm{o} i}\right)\left(\dot{\boldsymbol{p}}_{\mathrm{d} i}-\beta_{i} \Delta \boldsymbol{p}_{i}\right),
\end{align*}
$$

where $\dot{\overrightarrow{\boldsymbol{J}}}_{\varphi_{i}}^{+}\left(\dot{\boldsymbol{q}}_{\mathrm{o} i}\right), \dot{\hat{\boldsymbol{Q}}}_{i}\left(\dot{\boldsymbol{q}}_{\mathrm{o} i}\right)$ are defined in Gudiño Lau et al. (2004) and $\dot{\boldsymbol{q}}_{\mathrm{o} i} \triangleq \dot{\hat{\boldsymbol{q}}}_{i}-\boldsymbol{\Lambda}_{i} \boldsymbol{z}_{i}, \boldsymbol{r}_{i} \triangleq$ $\dot{\boldsymbol{q}}_{i}-\dot{\boldsymbol{q}}_{\mathrm{o} i}=\dot{\boldsymbol{z}}_{i}+\boldsymbol{\Lambda}_{i} \boldsymbol{z}_{i}$. Note that $\dot{\boldsymbol{p}}_{i}$ is still used since this value is known from (4). After some manipulation, it is possible to get
$\ddot{\hat{\boldsymbol{q}}}_{\mathrm{r} i}=\ddot{\boldsymbol{q}}_{\mathrm{r} i}+\boldsymbol{e}_{i}\left(\boldsymbol{r}_{i}\right)$,
where

$$
\begin{align*}
\boldsymbol{e}_{i}\left(\boldsymbol{r}_{i}\right) & \triangleq-\dot{\overline{\boldsymbol{Q}}}_{i}\left(\boldsymbol{r}_{i}\right)\left(\dot{\boldsymbol{q}}_{\mathrm{d} i}-\boldsymbol{\Lambda}_{i} \tilde{\boldsymbol{q}}_{i}+\boldsymbol{\Lambda}_{i} \boldsymbol{z}_{i}\right)  \tag{11}\\
& -\dot{\overline{\boldsymbol{J}}}_{\varphi_{i}}^{+}\left(\boldsymbol{r}_{i}\right)\left(\dot{\boldsymbol{p}}_{\mathrm{d} i}-\beta_{i} \Delta \boldsymbol{p}_{i}\right) .
\end{align*}
$$

The proposed controller is then given for each single input by
$\boldsymbol{\tau}_{i} \triangleq \boldsymbol{H}_{i}\left(\boldsymbol{q}_{i}\right) \ddot{\hat{\boldsymbol{q}}}_{\mathrm{r} i}+\boldsymbol{C}_{i}\left(\boldsymbol{q}_{i}, \dot{\boldsymbol{q}}_{\mathrm{r} i}\right) \dot{\boldsymbol{q}}_{\mathrm{r} i}$

$$
\begin{align*}
& +\boldsymbol{D}_{i} \dot{\boldsymbol{q}}_{\mathrm{r}_{i}}+\boldsymbol{g}_{i}\left(\boldsymbol{q}_{i}\right)  \tag{12}\\
& -\boldsymbol{K}_{\mathrm{R}_{i}}\left(\dot{\boldsymbol{q}}_{\mathrm{o} i}-\dot{\boldsymbol{q}}_{\mathrm{r} i}\right)-\boldsymbol{J}_{\varphi_{i}}^{\mathrm{T}}\left(\boldsymbol{q}_{i}\right) \boldsymbol{\lambda}_{\mathrm{d} i}
\end{align*}
$$

where $\boldsymbol{K}_{\mathrm{R} i} \in \mathbb{R}^{n_{i} \times n_{i}}$ is a diagonal positive definite matrix. By substituting (12) into (1), the closed loop dynamics becomes after many manipulation

$$
\begin{align*}
\boldsymbol{H}_{i}\left(\boldsymbol{q}_{i}\right) \dot{\boldsymbol{s}}_{i} & =-\boldsymbol{C}_{i}\left(\boldsymbol{q}_{i}, \dot{\boldsymbol{q}}_{i}\right) \boldsymbol{s}_{i}-\boldsymbol{K}_{\mathrm{DR}_{i}} \boldsymbol{s}_{i}  \tag{13}\\
& +\boldsymbol{K}_{\mathrm{R}_{i}} \boldsymbol{r}_{i}+\boldsymbol{J}_{\varphi_{i}}^{\mathrm{T}}\left(\boldsymbol{q}_{i}\right) \Delta \boldsymbol{\lambda}_{i} \\
& -\boldsymbol{C}_{i}\left(\boldsymbol{q}_{i}, \dot{\boldsymbol{q}}_{\mathrm{r} i}\right) \boldsymbol{s}_{i}+\boldsymbol{H}_{i}\left(\boldsymbol{q}_{i}\right) \boldsymbol{e}_{i}\left(\boldsymbol{r}_{i}\right)
\end{align*}
$$

where $\boldsymbol{K}_{\mathrm{DR}_{i}} \triangleq \boldsymbol{K}_{\mathrm{R}_{i}}+\boldsymbol{D}_{i}$. In order to get (13), Property 3 of Gudiño Lau et al. (2004) has been used.

## Observer definition

The proposed dynamics of the observer is given by
$\dot{\hat{\boldsymbol{q}}}_{i}=\dot{\hat{\boldsymbol{q}}}_{\mathrm{o} i}+\boldsymbol{\Lambda}_{i} \boldsymbol{z}_{i}+k_{\mathrm{d} i} \boldsymbol{z}_{i}$
$\ddot{\hat{\boldsymbol{q}}}_{\mathrm{o} i}=\ddot{\hat{\boldsymbol{q}}}_{\mathrm{r} i}+k_{\mathrm{d} i} \boldsymbol{\Lambda}_{i} \boldsymbol{z}_{i}$,
where $k_{\mathrm{d} i}$ is a positive constant. Since from (14) you have $\ddot{\hat{\boldsymbol{q}}}_{\mathrm{o} i}=\ddot{\hat{\boldsymbol{q}}}_{i}-\boldsymbol{\Lambda}_{i} \dot{\boldsymbol{z}}_{i}-k_{\mathrm{d} i} \dot{\boldsymbol{z}}_{i}$, (15) becomes $\dot{\boldsymbol{s}}_{i}=\dot{\boldsymbol{r}}_{i}+k_{\mathrm{d} i} \boldsymbol{r}_{i}+\boldsymbol{e}_{i}\left(\boldsymbol{r}_{i}\right)$, in view of (10). By multiplying both sides of $\dot{\boldsymbol{s}}_{i}$ by $\boldsymbol{H}_{i}\left(\boldsymbol{q}_{i}\right)$, and by taking into account (13), one gets

$$
\begin{align*}
\boldsymbol{H}_{i}\left(\boldsymbol{q}_{i}\right) \dot{\boldsymbol{r}}_{i} & =-\boldsymbol{C}_{i}\left(\boldsymbol{q}_{i}, \dot{\boldsymbol{q}}_{i}\right) \boldsymbol{r}_{i}-\boldsymbol{H}_{\mathrm{rd}} \boldsymbol{r}_{i}  \tag{16}\\
& +\boldsymbol{C}_{i}\left(\boldsymbol{q}_{i}, \boldsymbol{s}_{i}+\dot{\boldsymbol{q}}_{\mathrm{ri}}\right) \boldsymbol{r}_{i} \\
& -\boldsymbol{C}_{i}\left(\boldsymbol{q}_{i}, \boldsymbol{s}_{i}+2 \dot{\boldsymbol{q}}_{\mathrm{r} i}\right) \boldsymbol{s}_{i} \\
& -\boldsymbol{K}_{\mathrm{DR}_{i}} \boldsymbol{s}_{i}+\boldsymbol{J}_{\varphi_{i}}^{\mathrm{T}}\left(\boldsymbol{q}_{i}\right) \Delta \boldsymbol{\lambda}_{i} .
\end{align*}
$$

where $\boldsymbol{H}_{\mathrm{rd}_{i}} \triangleq k_{\mathrm{d} i} \boldsymbol{H}_{i}\left(\boldsymbol{q}_{i}\right)-\boldsymbol{K}_{\mathrm{R}_{i}}$, after some more manipulation now, let us define
$\boldsymbol{x}_{i} \triangleq\left[\begin{array}{ll}\boldsymbol{s}_{i}^{\mathrm{T}} & \boldsymbol{r}_{i}^{\mathrm{T}}\end{array}\right]^{\mathrm{T}}$,
as state for (13) and (16). The main idea of the control-observer design is to show that whenever $\left\|\boldsymbol{x}_{i}\right\|$ tends to zero, the tracking errors $\tilde{\boldsymbol{q}}_{i}, \dot{\tilde{\boldsymbol{q}}}_{i}, \Delta \boldsymbol{p}_{i}$, $\Delta \dot{\boldsymbol{p}}_{i}$ and $\Delta \boldsymbol{\lambda}_{i}$ and the observation errors $\boldsymbol{z}_{i}$ and $\dot{\boldsymbol{z}}_{i}$ will do it as well. The following lemma shows that this is indeed the case under some conditions.
Lemma 1 If $\boldsymbol{x}_{i}$ is bounded by $x_{\max _{i}}$ and tends to zero, then the following facts hold:
a) $\Delta \boldsymbol{p}_{i}$ and $\Delta \dot{\boldsymbol{p}}_{i}$ remain bounded and tend to zero.
b) $\tilde{\boldsymbol{q}}_{i}$ and $\dot{\tilde{\boldsymbol{q}}}_{i}$ remain bounded. Furthermore, if the bound $x_{\max _{i}}$ for $\left\|\boldsymbol{x}_{i}\right\|$ is chosen small enough so as to guarantee that $\left\|\tilde{\boldsymbol{q}}_{i}\right\| \leq \eta_{i}$ for all $t$, with $\eta_{i}$ a positive and small enough constant, then both $\tilde{\boldsymbol{q}}_{i}$ and $\dot{\tilde{\boldsymbol{q}}}_{i}$ will tend to zero as well.
c) If, in addition, the velocity vector $\dot{\boldsymbol{q}}_{i}$ is bounded, then $\Delta \boldsymbol{\lambda}_{i}$ will remain bounded and tend to zero.

The proof of Lemma 1 can be found in Appendix $A$. It is interesting to note that, if $\left\|\boldsymbol{x}_{i}\right\|$ is bounded by $x_{\max _{i}}$, then it is always possible to find a bound for $\boldsymbol{e}_{i}\left(\boldsymbol{r}_{i}\right)$ in (11) so that
$\left\|\boldsymbol{e}_{i}\left(\boldsymbol{r}_{i}\right)\right\| \leq M_{\mathrm{e} i}\left(x_{\max _{i}}\right)\left\|\boldsymbol{r}_{i}\right\|<\infty$.
Consider now the following function
$V_{i}\left(\boldsymbol{x}_{i}\right)=\frac{1}{2} \boldsymbol{x}_{i}^{\mathrm{T}} \boldsymbol{M}_{i} \boldsymbol{x}_{i}$,
where $\boldsymbol{M}_{i} \triangleq$ block diag $\left\{\boldsymbol{H}_{i}\left(\boldsymbol{q}_{i}\right), \quad \boldsymbol{H}_{i}\left(\boldsymbol{q}_{i}\right\}\right.$. The following theorem establishes the conditions for the controller-observer parameters to guarantee asymptotic stability.

Theorem 1 Consider the cooperative system dynamics given by (1), (2) and (4), in closed loop, with the control law (12) and the observer (14)(15), where $\boldsymbol{q}_{\mathrm{d} i}$ and $\boldsymbol{p}_{\mathrm{d} i}$ are the desired bounded joint and constrained positions, whose derivatives $\dot{\boldsymbol{q}}_{\mathrm{d} i}, \ddot{\boldsymbol{q}}_{\mathrm{d} i}, \dot{\boldsymbol{p}}_{\mathrm{d} i}$, and $\ddot{\boldsymbol{p}}_{\mathrm{d} i}$ are also bounded, and they all satisfy constraint (4). Consider also l given domains $\mathbb{D}_{i} \in \mathbb{R}^{n_{i}}$
$\mathbb{D}_{i}=\left\{\boldsymbol{x}_{i}:\left\|\boldsymbol{x}_{i}\right\| \leq x_{\text {max }_{i}}\right\}$,
for $i=1, \ldots, l$, with $x_{\max _{i}}$ small enough and
$x_{\max _{i}} \leq \frac{\eta_{i} \alpha_{i}}{\left(1+\sqrt{n_{i}}\right)}$
with $\alpha_{i} \triangleq k_{i}-\left|k_{i}-\beta_{i}\right|-\gamma_{i}, k_{i}$ and $\beta_{i}$ given in (7) and $\gamma_{i} \triangleq c_{0 i} L_{i}\left(v_{\mathrm{m} i}+\beta_{i} \eta_{i}\right)$, with $c_{0 i}$ and $L_{i}$ given in (assumption 2)-(5) and $\left\|\dot{\boldsymbol{q}}_{\mathrm{d} i}\right\| \leq v_{\mathrm{m} i} \forall t$. Then, every dynamic and error signal remains bounded and asymptotic stability of tracking, observation and force errors arise, i.e.

$$
\begin{gather*}
\lim _{t \rightarrow \infty} \tilde{\boldsymbol{q}}_{i}=\mathbf{0} \quad \underset{t \rightarrow \infty}{\lim _{t \rightarrow \infty} \dot{\tilde{\boldsymbol{q}}}_{i}=\mathbf{0} \quad} \quad \underset{t \rightarrow \infty}{\lim _{t \rightarrow \infty} \boldsymbol{z}_{i}=\mathbf{0}}  \tag{22}\\
\lim _{t \rightarrow \infty} \dot{\boldsymbol{z}}_{i}=\mathbf{0} \quad \lim _{t \rightarrow \infty} \Delta \boldsymbol{\lambda}_{i}=\mathbf{0}, \tag{23}
\end{gather*}
$$

if the following conditions are satisfied

$$
\begin{align*}
\lambda_{\min }\left(\boldsymbol{K}_{\mathrm{R}_{i}}\right) & \geq \mu_{1 i}+1+\delta_{i}  \tag{24}\\
k_{\mathrm{d} i} & \geq \frac{\lambda_{\max }\left(\boldsymbol{K}_{\mathrm{R}_{i}}\right)+\omega_{i}}{\lambda_{\mathrm{h} i}} \tag{25}
\end{align*}
$$

where $\omega_{i}=\mu_{2 i}+\gamma_{2 i}+\frac{1}{4}\left(\lambda_{\mathrm{D} i}+\mu_{3 i}+\mu_{4 i}+\gamma_{1 i}\right)^{2}+$ $\delta_{i}$, with $\delta_{i}$ a positive constant and $\mu_{1 i}, \mu_{2 i}, \mu_{3 i}$, $\mu_{4 i}, \gamma_{1 i}, \gamma_{2 i}$ and $\lambda_{\mathrm{D} i}$ defined in Appendix $B$.

The proof of the Theorem 1 can be found in Appendix B.

## 4 EXPERIMENTAL RESULTS

A test bed with two industrial robots is used. The robots are at the Laboratory for Robotics of the National University of Mexico. They are the A465 and A255 of CRS Robotics. Only the first three joints of each robot are used for the experiments. To implement control law (12), the motors dynamics has to be taken into account. Both robots own force sensors, so that one can verify whether the desired forces are being matched. The palm frame of the whole system is at the base of the robot A465, with its $x$-axis pointing towards the other manipulator. If the task consists in lifting the object and pushing with a desired force, then the constraints in Cartesian coordinates are simply given by
$\varphi_{i}=x_{i}-b_{i}=0$,
for $i=1,2$ and $b_{i}$ a positive constant. The desired trajectories are given by

$$
\begin{align*}
x_{\mathrm{d} 1} & =0.5530[\mathrm{~m}] \quad x_{\mathrm{d} 2}=0.8522[\mathrm{~m}]  \tag{27}\\
y_{\mathrm{d} 1,2} & =0.0095 \sin \left(\omega\left(t-t_{\mathrm{i}}\right)\right)[\mathrm{m}]  \tag{28}\\
z_{\mathrm{d} 1,2} & =\left(0.635+0.0095 \cos \left(\omega\left(t-t_{\mathrm{i}}\right)\right)\right.  \tag{29}\\
& -0.0095)[\mathrm{m}] .
\end{align*}
$$

These trajectories are valid from an initial time $t_{\mathrm{i}}$ to a final time $t_{\mathrm{f}}$, while $\omega$ is designed to satisfy $\omega\left(t_{\mathrm{i}}\right)=\omega\left(t_{\mathrm{f}}\right)=0$. The derivatives of $\omega$ are zero as well at $t_{\mathrm{i}}$ and $t_{\mathrm{f}}$. For the experiments it has been set $t_{\mathrm{i}}=20 \mathrm{~s}$ and $t_{\mathrm{f}}=70 \mathrm{~s}$. By choosing (27)-(28), the robots will make a circle each second in the $y-z$ plane. The desired pushing force is given by
$f_{\mathrm{d} x 1,2}=\left\{\begin{array}{lr}3.0(t-15)[\mathrm{N}] & 15 \leq t<25 \\ 30+10 \sin (6 \pi(t & \\ -25) / 40)[\mathrm{N}] & 25 \leq t \leq 65 \\ 30-3.0(t-65)[\mathrm{N}] & 65<t \leq 75\end{array}\right.$
and $f_{\mathrm{d} y 1,2}=f_{\mathrm{d} z 1,2}=0[\mathrm{~N}]$. The different control and observer parameters are $\boldsymbol{\Lambda}_{1}=21 \boldsymbol{I}, \boldsymbol{\Lambda}_{2}=$ $20 \boldsymbol{I}, \boldsymbol{K}_{\mathrm{R}_{1}}=80 \boldsymbol{I}, \boldsymbol{K}_{\mathrm{R}_{2}}=\operatorname{diag}\left\{\begin{array}{ccc}40 & 20 & 40\end{array}\right\}$, $k_{\mathrm{d} 1}=k_{\mathrm{d} 2}=12$.

The observer-controller scheme has been programmed in a PC computer, while the sampling time is $h=9 \mathrm{~ms}$. The experiment lasts 90s. The object is held at $t=15 \mathrm{~s}$. Before, the robots are in free movement and the control law (12) is used with the force part set to zero (i.e. $\boldsymbol{Q}_{i}=\boldsymbol{I}$ and $\left.\boldsymbol{J}_{\varphi_{i}}=\boldsymbol{O}\right)$. From $t=15 \mathrm{~s}$ to $t=20 \mathrm{~s}$ the object is lifted to the initial position to make the circles, while the desired pushing forces keep increasing. From $t=t_{\mathrm{i}}=20 \mathrm{~s}$ to $t=t_{\mathrm{f}}=70 \mathrm{~s}$ the robots are making the circles and the desired forces are sinus signals from $t=25 \mathrm{~s}$ to $t=65 \mathrm{~s}$. From $t=70 \mathrm{~s}$ to $t=75 \mathrm{~s}$ the object is put down and the desired forces diminish to zero. Finally, from $t=75 \mathrm{~s}$ to $t=90$ s the manipulators go back to their initial positions. The results for the tracking errors can be seen in Figure 1 in Cartesian coordinates. It can be appreciated that they are larger during the constrained motion. For the desired forces (30) the results can be considered good, although the main force in the $x$ direction shows some noise around the desired trajectory (see Figure 2). Figure 3 shows the observation errors. As can be appreciated, they are pretty good.

## 5 CONCLUSIONS

The position and force tracking control problem of cooperative robots with end effectors constrained on geometric surfaces and without velocity-force measurements is considered in this paper by using the joint-space orthogonalization scheme. The control law is a decentralized approach which takes into account motion constraints rather than the held object dynamics. By assuming that fingers dynamics are well known, the crucial point of this work is to show that our controller does not need any velocity-force


Fig. 1: Tracking errors in Cartesian coordinates.


Fig. 2: Force measurements of robots A465 and A255. a) $F_{x_{1}}$. b) $F_{y_{1}}$. c) $F_{z_{1}}$. a) $F_{x_{2}}$. b) $F_{y_{2}}$. c) $F_{z_{2}}$. - measured. - - desired.
feedback. A linear observer for each finger is proposed to estimate joint velocities which does not require any knowledge of the robots dynamics. Regarding the force control, our scheme only uses a feedforward of the desired force. Despite the fact that the stability analysis is complex, the controller and specially the observer are not. Experimental results have been carried out to test the proposed approach. Since both robots own force sensors, it was possible to check out that there was a good matching of real and desired forces. Also, the overall outcomes can be


Fig. 3: Observation errors. a) $z_{11}$. b) $z_{12}$. c)

$$
z_{13} . \text { d) } z_{21} . \text { e) } z_{22} . \text { f) } z_{23} .
$$

considered good as well.

## A Proof of Lemma 1

In this appendix, item c) of Lemma 1 is proven. For a proof of items a) and b) see Gudiño Lau et al. (2004). Note that one only has to set $\boldsymbol{x}_{i}^{\mathrm{T}}=\left[\begin{array}{ll}\boldsymbol{s}_{i}^{\mathrm{T}} & \boldsymbol{r}_{i}^{\mathrm{T}}\end{array}\right]^{\mathrm{T}}$ and $\boldsymbol{\xi}_{i}=\boldsymbol{O}$, where $\boldsymbol{\xi}_{i}$ is the gain for the force feedback $\Delta \boldsymbol{F}_{i}$, which is not used here.
c) When $\left\|\boldsymbol{x}_{i}\right\|$ is bounded and tends to zero, $\Delta \boldsymbol{\lambda}_{i}$ does not necessarily do it nor remains bounded. In order to prove that, one may use the fact that $\boldsymbol{J}_{\varphi_{i}}\left(\boldsymbol{q}_{i}\right) \boldsymbol{s}_{i}=\Delta \dot{\boldsymbol{p}}_{i}+\beta_{i} \Delta \boldsymbol{p}_{i}=$ 0 . The last equality is valid since constraint (4) must be satisfied. Thus you have $\boldsymbol{J}_{\varphi_{i}}\left(\boldsymbol{q}_{i}\right) \dot{s}_{i}+\dot{\boldsymbol{J}}_{\varphi_{i}}\left(\boldsymbol{q}_{i}\right) \boldsymbol{s}_{i}=0$, and from (13) one gets

$$
\begin{align*}
\Delta \boldsymbol{\lambda}_{i} & =-\left(\boldsymbol{J}_{\varphi_{i}}\left(\boldsymbol{q}_{i}\right) \boldsymbol{H}_{i}^{-1}\left(\boldsymbol{q}_{i}\right) \boldsymbol{J}_{\varphi_{i}}^{T}\left(\boldsymbol{q}_{i}\right)\right)^{-1}  \tag{A.1}\\
& \cdot\left(\dot{\boldsymbol{J}}_{\varphi_{i}}\left(\boldsymbol{q}_{i}\right) \boldsymbol{s}_{i}+\boldsymbol{J}_{\varphi_{i}}\left(\boldsymbol{q}_{i}\right) \boldsymbol{H}_{i}^{-1}\left(\boldsymbol{q}_{i}\right) \boldsymbol{h}_{i}(\boldsymbol{t})\right)
\end{align*}
$$

with

$$
\begin{align*}
\boldsymbol{h}_{i} & =\boldsymbol{H}_{i}\left(\boldsymbol{q}_{i}\right) \boldsymbol{e}_{i}\left(\boldsymbol{r}_{i}\right)-\boldsymbol{C}_{i}\left(\boldsymbol{q}_{i}, \dot{\boldsymbol{q}}_{i}\right) s_{i}  \tag{A.2}\\
& +\boldsymbol{K}_{\mathrm{R}_{i}} \boldsymbol{r}_{i}-\boldsymbol{C}_{i}\left(\boldsymbol{q}_{i}, \dot{\boldsymbol{q}}_{\mathrm{r} i}\right) s_{i}-\boldsymbol{K}_{\mathrm{DR}_{i}} \boldsymbol{s}_{i} .
\end{align*}
$$

Because of the assumption of the boundedness of $\dot{\boldsymbol{q}}$ and $\boldsymbol{x}_{i}, \Delta \boldsymbol{\lambda}_{i}$ must be bounded as well from (A.1). Furthermore, if $\left\|\boldsymbol{x}_{i}\right\| \rightarrow 0$ then
$\Delta \boldsymbol{\lambda}_{i} \boldsymbol{\rightarrow} \mathbf{0}$. Finally, note that from (13) we can aditionally conclude that $\dot{s}_{i}$ is bounded and tends to zero.

## B Proof of Theorem 1

We just have to find domains $\mathbb{D}_{i}$ for which each $V_{i}\left(\boldsymbol{x}_{i}\right)$ in (19) satisfies $\dot{V}_{i}\left(\boldsymbol{x}_{i}\right)<0$ in $\mathbb{D}_{i}-\{\mathbf{0}\}$. Note that $V_{i}\left(\boldsymbol{x}_{i}\right)$ is positive definite in $\mathbb{R}^{n_{i}}$. In doing so, one can prove that $\boldsymbol{x}_{i} \rightarrow \mathbf{0}$ for all $i$. Then, Lemma 1 can be employed to analyze the behavior of the different error signals. Based on the discussion given in Appendix A, we define each domain $\mathbb{D}_{i}$ as in (20), where $x_{\text {max }_{i}}$ is chosen $x_{\max _{i}} \leq \frac{\eta_{i} \alpha_{i}}{1+\sqrt{n_{i}}}$. See Appendix I of Gudiño Lau et al. (2004) for details. Note that $x_{\max _{i}}$ cannot be done arbitrarily large. In $\mathbb{D}_{i}$ one can define

$$
\begin{align*}
& \mu_{1 i} \triangleq \max _{\left\|\boldsymbol{x}_{i}\right\| \leq x_{\text {max }}}\left\|\boldsymbol{C}_{i}\left(\boldsymbol{q}_{i}, \dot{\boldsymbol{q}}_{\mathrm{r} i}\right)\right\|  \tag{B.1}\\
& \mu_{2 i} \triangleq \max _{\left\|\boldsymbol{x}_{i}\right\| \leq x_{\text {max }}}\left\|\boldsymbol{C}_{i}\left(\boldsymbol{q}_{i}, \boldsymbol{s}_{i}+\dot{\boldsymbol{q}}_{\mathrm{r} i}\right)\right\|  \tag{B.2}\\
& \mu_{3 i} \triangleq \max _{\left\|\boldsymbol{x}_{i}\right\| \leq x_{\max }}\left\|\boldsymbol{C}_{i}\left(\boldsymbol{q}_{i}, s_{i}+2 \dot{\boldsymbol{q}}_{\mathrm{r} i}\right)\right\|  \tag{B.3}\\
& \mu_{4 i} \triangleq M_{\mathrm{ei} i}\left(x_{\text {max }_{i}}\right) \lambda_{\mathrm{H} i}  \tag{B.4}\\
& \lambda_{\mathrm{D} i} \stackrel{\Delta}{ } \lambda_{\text {max }}\left(\boldsymbol{D}_{i}\right)  \tag{B.5}\\
& c_{2 i}=\left\|\left[\boldsymbol{J}_{\varphi_{i}}\left(\boldsymbol{q}_{i}\right) \boldsymbol{H}_{i}^{-1}\left(\boldsymbol{q}_{i}\right) \boldsymbol{J}_{\varphi_{i}}^{T}\left(\boldsymbol{q}_{i}\right)\right]^{-1}\right\|  \tag{B.6}\\
& \left\|\dot{\boldsymbol{J}}_{\varphi_{i}}\left(\boldsymbol{q}_{i}\right)\right\| \leq \alpha_{1 i}\left\|\boldsymbol{s}_{i}\right\|+\alpha_{2 i}\left\|\dot{\boldsymbol{q}}_{\mathrm{r} i}\right\|  \tag{B.7}\\
& \left\|\dot{\boldsymbol{q}}_{\mathrm{r} i}\right\| \leq v_{\mathrm{m} i}+k_{i} \eta_{i}+\sqrt{n_{i}} x_{\max _{i}}  \tag{B.8}\\
& \sigma_{\mathrm{H} i}=\max _{\forall \boldsymbol{q}_{i} \in \mathbb{R}^{n_{i}}} \lambda_{\max }\left(\boldsymbol{H}_{i}^{-1}\right)  \tag{B.9}\\
& \gamma_{1 i} \triangleq c_{1 i} c_{2 i}\left(\alpha_{1 i} x_{\max _{i}}+\alpha_{2 i} \alpha_{3 i}\right)  \tag{B.10}\\
& +c_{1 i}{ }^{2} c_{2 i} \sigma_{\mathrm{H} i}\left(\mu_{3 i}+\lambda_{\max _{i}}\left(\boldsymbol{K}_{\mathrm{DR}_{i}}\right)\right) \\
& \gamma_{2 i} \triangleq c_{1 i}{ }^{2} c_{2 i} \sigma_{H i} \lambda_{\max _{i}}\left(\boldsymbol{K}_{\mathrm{R}_{i}}\right)  \tag{B.11}\\
& +c_{1 i}{ }^{2} c_{2 i} M_{\mathrm{e} i} \\
& \left\|\boldsymbol{r}_{i}^{\mathrm{T}} \boldsymbol{J}_{\varphi_{i}}^{\mathrm{T}}\left(\boldsymbol{q}_{i}\right) \Delta \boldsymbol{\lambda}_{i}\right\| \leq \gamma_{1 i}\left\|\boldsymbol{s}_{i}\right\|\left\|\boldsymbol{r}_{i}\right\|  \tag{B.12}\\
& +\gamma_{2 i}\left\|\boldsymbol{r}_{i}\right\|^{2},
\end{align*}
$$

where $\alpha_{1 i}, \alpha_{2 i}$ are positive constants, $M_{\mathrm{e} i}$ is given in (18), $\left\|\dot{\boldsymbol{q}}_{\mathrm{d} i}\right\| \leq v_{\mathrm{m} i} \forall \mathrm{t}$, and $\eta_{i}$ small enough. The next step is to compute the derivative of the Lyapunov function candidate in (19) along (13) and (16), which can be simplified to

$$
\begin{align*}
\dot{V}_{i}\left(\boldsymbol{x}_{i}\right) \leq & -\lambda_{\min }\left(\boldsymbol{K}_{\mathrm{R}_{i}}\right)\left\|\boldsymbol{s}_{i}\right\|^{2}-k_{\mathrm{d}_{i}} \lambda_{\mathrm{h}_{i}}\left\|\boldsymbol{r}_{i}\right\|^{2}  \tag{B.13}\\
& +\lambda_{\max }\left(\boldsymbol{K}_{\mathrm{R}_{i}}\right)\left\|\boldsymbol{r}_{i}\right\|^{2}+\gamma_{2 i}\left\|\boldsymbol{r}_{i}\right\|^{2} \\
& +\mu_{1 i}\left\|\boldsymbol{s}_{i}\right\|^{2}+\mu_{2 i}\left\|\boldsymbol{r}_{i}\right\|^{2}
\end{align*}
$$

$$
+\left(\lambda_{\mathrm{D} i}+\mu_{3 i}+\mu_{4 i}+\gamma_{1 i}\right)\left\|\boldsymbol{s}_{i}\right\|\left\|\boldsymbol{r}_{i}\right\|
$$

from (B.1)-(B.12), since we are only interested in the behavior of $\dot{V}_{i}\left(\boldsymbol{x}_{i}\right)$ for $\boldsymbol{x}_{i}$ in $\mathbb{D}_{i}$. Consider $\lambda_{\min }\left(\boldsymbol{K}_{\mathrm{R}_{i}}\right)$ in (24) and $k_{\mathrm{d} i}$ in (25), so that
$\dot{V}_{i}\left(\boldsymbol{x}_{i}\right) \leq-\delta_{i}\left\|\boldsymbol{x}_{i}\right\|^{2}$.
Then, one concludes that $\boldsymbol{x}_{i} \rightarrow \mathbf{0}$. Now, from definition of $\boldsymbol{r}_{i}$ one has directly $\lim _{t \rightarrow \infty} \boldsymbol{z}_{i}=\mathbf{0}$ and $\lim _{t \rightarrow \infty} \dot{\boldsymbol{z}}_{i}=\mathbf{0}$. Furthermore, one has $\left\|\boldsymbol{x}_{i}\right\| \leq x_{\max _{i}}$ and thus $\left\|\tilde{\boldsymbol{q}}_{i}\right\| \leq \eta_{i}$ (from the discussion in Appendix I of Gudiño Lau et al. (2004)). Thus, from Lemma 1, a) and b), we get $\lim _{t \rightarrow \infty} \Delta \dot{\boldsymbol{p}}_{i}=\mathbf{0}$, $\lim _{t \rightarrow \infty} \Delta \boldsymbol{p}_{i}=\mathbf{0}, \lim _{t \rightarrow \infty} \dot{\tilde{\boldsymbol{q}}}_{i}=\mathbf{0}, \lim _{t \rightarrow \infty} \tilde{\boldsymbol{q}}_{i}=\mathbf{0}$. To applied c) of Lemma 1, we only need to show that $\dot{\boldsymbol{q}}_{i}$ is bounded. This is certainly the case because $\dot{\tilde{\boldsymbol{q}}}_{i}$ and $\dot{\boldsymbol{q}}_{\mathrm{d} i}$ are bounded. Thus we get $\lim _{t \rightarrow \infty} \Delta \boldsymbol{\lambda}_{i}=\mathbf{0}$. Finally, the stability of the whole system can be proven using $V=\sum_{i=1}^{l} V_{i}\left(\boldsymbol{x}_{i}\right)$.

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[^0]:    ${ }^{1}$ This work is based on research supported by the DGAPA-UNAM under grants IN119003 and IX116804 and by the CONACYT.

